positional tolerance of 0.0005 inch, the limit of motional error is 0.066°. From Fig. 7, the errors associated with the waveguide tolerances are 0.00068° per degree for WR-90 and 0.00032 for WR-112. For 60°, there is a limit of waveguide dimensional error of 0.06°. From Fig. 9, for a 0.1° short circuit misalignment the total phase error is 0.0014° per degree for WR-90 and 0.0012° per degree for WR-119. For 60°, this is a limit of error of 0.156°. The total limit of error from all these sources is then 0.335°. If a single phase shifter was built to these specifications in WR-90, the total error would be 0.437°.

If, however, for the same system specifications the measured angle was 90°, the total error for the differential system would be 0.585° as compared to 0.464° for the single system. In this example, the reduction of positional error is more than offset by an increase in tuning error.

Conclusions

Although the instrument described in this paper provides for a smaller short circuit positional error as compared with that for a single phase shifter, the combined tuning and waveguide dimensional errors are increased. For certain measured angles, the over-all increase of error will more than offset the positional error reduction achieved through the use of a differential system.

This instrument should be particularly useful where small changes of phase angle are involved because all the errors, other than the short circuit positional error, are proportional to the measured angle.

Acknowledgment

The author extends appreciation to W. A. Downing for computing and graphically presenting the data in Fig. 4.

Maximum Bandwidth Performance of a Nondegenerate Parametric Amplifier with Single-Tuned Idler Circuit

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Summary—**The single-tuned bandwidth and limiting flat bandwidth of a nondegenerate reflection-type diode parametric amplifier is calculated. The amplifier has a broad-banding filter structure in the signal circuit and a single-tuned idler circuit. An experimental low-noise, wide-band *L*-band amplifier is described, and measurement results are presented. The amplifier has a triple-tuned signal circuit and a single-tuned idler circuit and is pumped at 11.3 Gc. A nearly flat bandwidth of 23 per cent at 7 db gain and an effective input noise temperature of 70°K at room temperature ambient and of 29°K at liquid nitrogen (77°K) ambient has been obtained.**

I. Introduction

Several authors have discussed the design and gain-bandwidth limitation of a nondegenerate diode parametric amplifier of the reflection type employing broad-banding filters in signal and idler circuit.[1–3] It has not been established, however, whether it is necessary to have broad band-pass filters in both circuits. In this paper the gain-bandwidth limitation of a similar amplifier employing a band-pass filter in the signal circuit only (and a single-tuned filter in the idler circuit) is investigated theoretically and experimentally, for the following two reasons. 1) For low microwave frequency operation, the idler frequency can be made so much higher than the signal frequency that the absolute bandwidth of the idler circuit is much broader than that

Manuscript received March 16, 1964; revised April 20, 1964. This paper was submitted posthumously. Correspondence concerning it should be addressed to R. S. Engelbrecht, Bell Telephone Laboratories, Inc., Murray Hill, N. J.

[1] R. Aron, "Gain bandwidth relations in negative resistance amplifiers," Proc. IRE (*Correspondence*), vol. 49, pp. 355–356; January, 1961.

[2] E. S. Kuh and M. Fukada, "Optimum synthesis of wide-band parametric amplifiers and converters," IRE Trans. on Circuit Theory, vol. CT-9, pp. 410–415; December, 1961.

[3] B. T. Henoch, "A new method for designing wide-band parametric amplifiers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 62–72; January, 1963.

of the signal circuit. Hence the limiting bandwidth of the amplifier may not be significantly different from that of the amplifier with an optimally compensated idler circuit. The single-tuned idler circuit simplifies the amplifier construction. 2) The diode resistance can be used advantageously as the only idler load, and this minimizes the effective input noise temperature of the amplifier at a fixed idler frequency.

The gain-bandwidth limitation of an amplifier with single-tuned idler circuit has been obtained from that of a cavity-maser.[4] The optimum idler load resistance for maximum limiting flat bandwidth is derived. For comparison, the bandwidth obtained with single tuning in both the signal and idler circuit is also calculated.

As has been pointed out by Matthaei,[5] it is difficult to resonate the diode independently at the signal and idler frequency. An investigation shows that in addition to inductances at least one external capacitance is required as a tuning element. The bandwidth limitation due to this capacitance is calculated. Taking into account the internal diode lead inductance, it is shown that in general the optimum location of this capacitance is parallel to the diode, which also leads to a simple construction of the amplifier.

The exact calculation of the bandwidth of a parametric amplifier is complicated, and usually some approximations are necessary. For the calculation of the bandwidth limitation it has been assumed in this paper that the diode is lossless at the signal frequency. This assumption will seldom lead to large errors in practice, as will be shown. No "high gain" approximation has been made, since an amplifier may be operated at fairly low gain in order to increase the bandwidth.

Finally, an experimental triple-tuned L-band amplifier is described and some measurement results are presented.

II. Input Impedance of Single-Tuned Amplifier

Assume a variable capacitance having a time variation

$$C(t) = C_0/(1 + \gamma \cos \omega_p t) \tag{1}$$

where C_0 is the mean value of the capacitance, γ a capacitance modulation factor, and ω_p the angular pump frequency. Let $C(t)$ be connected to an impedance of value $\overline{Z}_2$ at an angular idler frequency ω_2, and open circuited at other frequencies. At a signal frequency $\omega_1 = \omega_p - \omega_2$ the impedance of $C(t)$ is then

$$\overline{Z}_c = \frac{1}{j\omega_1 C_0} - \frac{\gamma^2}{4\omega_1\omega_2 C_0^2 \overline{Z}_2^*} \tag{2}$$

[4] R. L. Kyhl, R. A. McFarlane, and M. W. P. Strandberg, "Negative L and C in solid-state masers," Proc. IRE, vol. 50, pp. 1608–1623; July, 1962.

[5] G. L. Matthaei, "A study of the optimum design of wide-band parametric amplifiers and up-convertors," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 23–38; January, 1961.

where the asterisk denotes the complex conjugate.[6]

In order to utilize the negative real part of $\overline{Z}_c$ effectively for amplification, C_0 must be simultaneously resonated at ω_1 and ω_2. Assume that series tuning is used as indicated in Fig. 1, in which F_1 and F_2 are ideal filters such that F_1 is a short circuit at ω_1 and an open circuit at other frequencies, F_2 is a short circuit at ω_2 and an open circuit at other frequencies, and R_L is an idler load resistance. Let the reactance X of a tuning element T, common to signal and idler circuit, be such that

$$\omega_1 C_0 X(\omega_1) = 1; \qquad \omega_2 C_0 X(\omega_2) = 1. \tag{3}$$

To a first order T can be generally represented as a function of frequency in the vicinity of ω_1 by a series connection of an L' and C', in the vicinity of ω_2 by L'' and C''. We now define so-called slope factors d_1 and d_2 which, near ω_1 and ω_2, represent the ratio of the reactance slope of C_0 plus T to the reactance slope of C_0 alone,

$$d_1 = 1 + \frac{C_0}{C'} = \frac{1}{2} + \frac{1}{2}\omega_1^2 C_0 \left(\frac{dX}{d\omega}\right)_{\omega=\omega_1} \tag{4}$$

$$d_2 = 1 + \frac{C_0}{C''} = \frac{1}{2} + \frac{1}{2}\omega_2^2 C_0 \left(\frac{dX}{d\omega}\right)_{\omega=\omega_2}. \tag{5}$$

The input impedance $\overline{Z}_A$ of the so-called single-tuned amplifier circuit at a frequency $\omega = \omega_1 + \Delta\omega$ (for small $\Delta\omega$), using (2) with $\overline{Z}_2 = R_L + jX(\omega_2) + 1/j\omega_2 C_0$, and neglecting the variation of the product of the instantaneous signal and idler frequencies, is approximately

$$\overline{Z}_A = \frac{2jd_1\Delta\omega}{\omega_1^2 C_0} - \frac{\gamma^2}{4\omega_1\omega_2 C_0^2 (R_L + 2jd_2\Delta\omega/\omega_2^2 C_0)} \cdot \tag{6}$$

The errors of (6), and the errors of the following bandwidth calculations based upon (6), depend on the ratio ω_2/ω_1 and the tuning element T. For the example of Section VIII, the representation (6) is quite accurate in a 20 per cent frequency band.

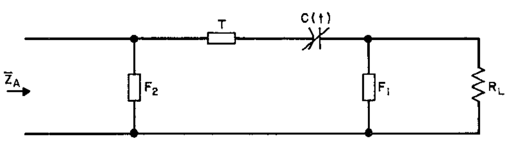


Fig. 1—Single-tuned parametric amplifier circuit. F = short circuit at ω_1, open at other frequency, F_2 = short circuit at ω_2, open at other frequency.

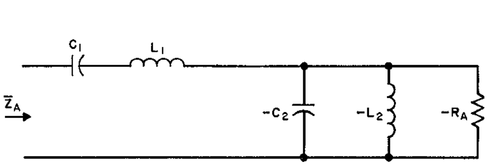


Fig. 2—Equivalent circuit of single-tuned parametric amplifier near resonance.

[6] B. J. Robinson, "Theory of variable-capacitance parametric amplifiers," Proc. IEE, Monograph 480E; November, 1961.

Alternatively, $\overline{Z}_A$ can be written

$$\overline{Z}_A = R_A \left\{ 2j \frac{\Delta\omega}{\omega_1} Q_1 - \frac{1}{1 + 2j \frac{\Delta\omega}{\omega_1} Q_2} \right\} \tag{7}$$

which suggests an equivalent circuit consisting of a positive and negative resonant circuit as shown in Fig. 2, where

$$\omega_1^2 = \frac{1}{L_1C_1} = \frac{1}{L_2C_2} \tag{8}$$

$$R_A = \frac{\gamma^2}{4\omega_1\omega_2C_0^2R_L} \tag{9}$$

$$Q_1 = \frac{1}{\omega_1C_1R_A} = \frac{4}{\gamma^2}\omega_2 d_1 C_0 R_L \tag{10}$$

$$Q_2 = \omega_1C_2R_A = \frac{\omega_1}{\omega_2}\frac{d_2}{\omega_2C_0R_L}\,. \tag{11}$$

We define

$$q = \sqrt{Q_2/Q_1} = \frac{\gamma}{2\omega_1C_0R_L}\sqrt{\frac{d_2}{d_1}\left(\frac{\omega_1}{\omega_2}\right)^3} \tag{12}$$

$$Q_M = \sqrt{Q_1Q_2} = \frac{2}{\gamma}\sqrt{d_1d_2\frac{\omega_1}{\omega_2}}\,. \tag{13}$$

III. Gain-Bandwidth Relations

Assume that the single-tuned amplifier is connected via a lossless coupling network N to an ideal circulator. The equivalent circuit is given in Fig. 3.

Consider first the frequency response of the single-tuned amplifier. If N consists of an ideal transformer, the absolute value of the voltage gain is given by

$$g(\omega) = \left| \frac{n^2Z_0 - \overline{Z}_A}{n^2Z_0 + \overline{Z}_A} \right| \tag{14}$$

in which n^2Z_0 is the transformed impedance of the circulator, and $\overline{Z}_A$ is given by (7). The 3-db bandwidth B is calculated from the following equation:

$$\left(\frac{BQ_M}{\omega_1}\right)^4 + \left(\frac{BQ_M}{\omega_1}\right)^2 \left\{2 + \frac{1}{q^2} + \left(\frac{g+1}{g-1}\right)^2 q^2\right\} - \frac{4g^2}{(g-1)^2(g^2-2)} = 0 \tag{15}$$

where g is the voltage gain at resonance. Assuming Q_M is constant, the bandwidth is a maximum for

$$q = q_{\text{opt}} = \sqrt{\frac{g-1}{g+1}}\,. \tag{16}$$

This maximum exists because for large q the bandwidth is limited by the idler circuit (Q_2) and for small q by the signal circuit (Q_1). For large gain, (15) reduces to

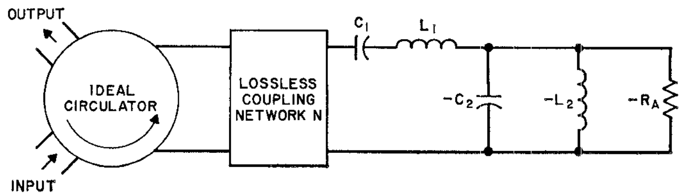


Fig. 3—Circuit used for bandwidth calculation.

$$\frac{BQ_M}{\omega_1} g = \frac{2}{\sqrt{2 + q^2 + 1/q^2}} \tag{17}$$

which has a maximum value of 1 for $q = 1$.

The bandwidth of the amplifier can be increased if N contains a band-pass filter consisting of resonators connected alternately in series and in parallel as shown in Fig. 9. However with an infinite number of resonators the maximum bandwidth remains finite. From a contour integration of $g(\omega)$ in the complex frequency plane follows

$$\frac{1}{\pi\omega_1}\int_0^\infty \ln g(\omega)d\omega = -\frac{1}{Q_2} + k - \sum_i l_i \tag{18}$$

$$\frac{1}{\pi\omega_1^3}\int_0^\infty (\omega-\omega_1)^2 \ln g(\omega)d\omega = \frac{1}{12}\frac{1}{Q_2^3} + \frac{1}{4}\frac{1}{Q_1Q_2^2} - \frac{1}{12}k^2 + \frac{1}{12}\sum l_i^3, \tag{19}$$

where k and l_i are respectively a pole and zeros of $g(\omega)$ in the right half of the complex frequency plane.[4] Obviously the maximum flat bandwidth, which is usually of interest in practice, is obtained if the voltage gain $g(\omega)$ is constant equal to g in a frequency band B and unity outside B, in which case the integrals can be evaluated directly. Elimination of k from (18) and (19), where we let $l_i = 0$ to maximize B,[4] gives

$$\frac{1}{3}q^2\left(\frac{BQ_M}{\omega_1}\frac{\ln g}{\pi}\right)^3\left(1 + \frac{\pi^2}{\ln^2 g}\right) + q\left(\frac{BQ_M}{\omega_1}\frac{\ln g}{\pi}\right)^2 + \left(\frac{BQ_M}{\omega_1}\frac{\ln g}{\pi}\right) - q = 0. \tag{20}$$

As in the previous case an optimum value of q for maximum limiting flat bandwidth at constant Q_M is obtained from (20) as follows:

$$q_{\text{opt}} = \sqrt{\frac{1 + 2\sqrt{(1 + \pi^2/\ln^2 g)/3}}{(1 + \pi^2/\ln^2 g)/3}}\,. \tag{21}$$

The corresponding maximum limiting flat bandwidth is given by

$$\left(\frac{BQ_M}{\omega_1}\right)_{\max} \ln g = \frac{\pi}{\sqrt{1 + 2\sqrt{(1 + \pi^2/\ln^2 g)/3}}}\,. \tag{22}$$

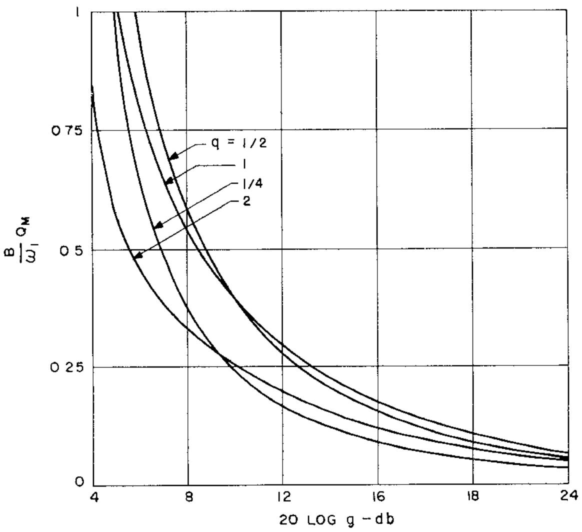


Fig. 4—Single-tuned 3-db bandwidth as function of gain.

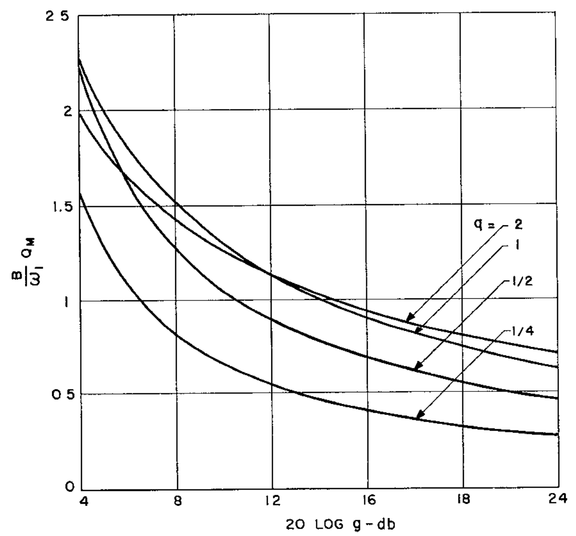


Fig. 5—Limiting flat bandwidth as function of gain.

As the gain becomes very high, the solution of (20) approaches

$$\frac{BQ_M}{\omega_1} \ln g = \frac{\pi(\sqrt[3]{1 + 3q^2} - 1)}{q} \tag{23}$$

which has a maximum value of 2.15 for $q = 2.54$.

In Figs. 4, 5 and 6 the single-tuned and limiting flat bandwidth according to (15) and (20) are plotted as a function of gain with q as a parameter.

With the aid of Figs. 4, 5 and 6 the single-tuned and limiting flat relative bandwidth of the amplifier can be determined as function of the parameters ω_2/ω_1, R_L and g and the usually fixed parameters γ, C_0, d_1 and d_2. The maximum bandwidth at arbitrary R_L is inversely proportional to Q_M, which requires that the capacitance modulation factor γ, the idler to signal frequency ratio ω_2/ω_1 and the inverse of the product of the slope factors

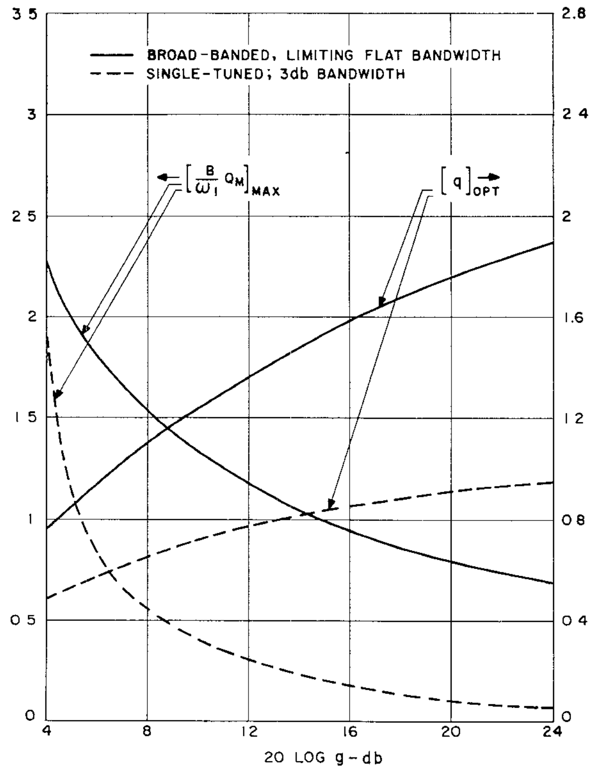


Fig. 6—Maximum bandwidth at constant Q_M, and optimum q, as function of gain.

d_1 and d_2 are as large as possible, and that R_L is equal to

$$(R_L)_{\text{opt}} = \frac{1}{q_{\text{opt}}} \frac{\gamma}{2\omega_1 C_0} \sqrt{\frac{d_2}{d_1}\left(\frac{\omega_1}{\omega_2}\right)^3} \tag{24}$$

where q_{opt} is given in Fig. 6.

In a similar way the limiting flat bandwidth of an amplifier having band-pass filters both in signal and idler circuit and tuned to the signal and idler frequencies, respectively, can be derived. For a lossless variable capacitance, at moderately high gain it is given by

$$\frac{BQ_M}{\omega_1} \ln 2g = \frac{\pi}{2} \tag{25}$$

with $d_1 = d_2 = 1$.[1] Comparison of (25) and (22), assuming in the latter case according to (28) $d_1 d_2 = 4$, shows that the maximum limiting flat bandwidth of an amplifier with filters only in the signal circuit is about 75 per cent of that of an amplifier with filters in both signal and idler circuit. In practice, this relatively small difference usually does not justify the complexity of using broadbanding filters in the idler circit.

Although a flat bandwidth cannot be obtained with a finite filter network N, (20) shows what can approximately be accomplished with broad-band operation. No exact information is available for the maximum obtainable bandwidth for finite N, and the design of N. Some calculations for N containing a single resonator filter have been made.[4,7] The previous results suggest that the optimum value of q for finite N is an average between the optimum values of q for maximum single-tuned bandwidth and maximum limiting flat bandwidth.

IV. Circuit Limitations

According to Section III the maximum bandwidth at arbitrary R_L is inversely proportional to $\sqrt{d_1 d_2}$, which requires that d_1 and d_2 be as small as possible. From (3), (4) and (5) it follows that if $\omega_1 = \omega_2$ the minimum value of d_1 and d_2 is 1, and is obtained if T is an inductance (degenerate amplifier).

Assuming $\omega_1 < \omega_2$, then according to (3) the reactance X of T has to satisfy $X(\omega_1) > 0$, $X(\omega_2) > 0$, and $X(\omega_1) > X(\omega_2)$. This requires that the T consists of at least three elements, C_p, L_p, and $(L_0 + L_s)$ as indicated in Fig. 7. We assume that L_0, representing the usual parasitic "lead" inductance of the variable capacitance, is fixed and that $\omega_1 < \omega_D$ where $\omega_D = 1/\sqrt{L_0 C_0}$ is the angular self-resonance frequency of the variable capacitance.

With $X = \omega(L_0 + L_s) + \omega L_p/(1 - \omega^2 L_p C_p)$ it follows from (3), (4) and (5) that

$$\frac{C_0}{C_p} = \left\{\frac{\omega_2^2}{\omega_D^2}\left(1 + \frac{L_s}{L_0}\right) - 1\right\} \cdot \left\{1 - \frac{\omega_1^2}{\omega_D^2}\left(1 + \frac{L_s}{L_0}\right)\right\} \quad (26)$$

$$d_1 - 1 = \frac{1}{d_2 - 1} = \frac{1 - \dfrac{\omega_1^2}{\omega_D^2}\left(1 + \dfrac{L_s}{L_0}\right)}{\dfrac{\omega_2^2}{\omega_D^2}\left(1 + \dfrac{L_s}{L_0}\right) - 1}, \quad (27)$$

from which we obtain

$$d_1 d_2 \geq 4. \quad (28)$$

For minimum $d_1 d_2$ at $\omega_2^2 < 2\omega_D^2 - \omega_1^2$ it follows from (27) that $(L_s/L_0)_{\text{opt}} = 2\omega_D^2/(\omega_1^2 + \omega_2^2) - 1$; for minimum $d_1 d_2$ at $\omega_2^2 > 2\omega_D^2 - \omega_1^2$, we obtain $(L_s)_{\text{opt}} = 0$. Substitution of (27) in (13) at optimum L_s/L_0 gives (with $L_s \geq 0$)

$$Q_M = \frac{4}{\gamma}\sqrt{\frac{\omega_1}{\omega_2}}, \qquad \omega_2^2 < 2\omega_D^2 - \omega_1^2 \quad (29)$$

$$Q_M = \frac{2}{\gamma}\sqrt{\frac{\omega_1}{\omega_2}} \frac{\omega_2^2 - \omega_1^2}{\sqrt{(\omega_2^2 - \omega_D^2)(\omega_D^2 - \omega_1^2)}}, \qquad \omega_2^2 > 2\omega_D^2 - \omega_1^2. \quad (30)$$

It follows that due to ω_D, Q_M has a minimum value as function of ω_2.

[7] J. Kliphuis, "C-band nondegenerate parametric amplifier with 500-Mc bandwidth," Proc. IRE (*Correspondence*), vol. 49, p. 961; May, 1961.

In order to simplify the discussion we assume in the following

$$\frac{\omega_1^2}{\omega_D^2} \ll 1 \quad \text{and} \quad \frac{\omega_1^2}{\omega_2^2} \ll 1. \quad (31)$$

The resulting approximate values of Q_M, d_1 and d_2 at optimum L_s are plotted in Fig. 8. Following from (30) Q_M is a minimum at approximately $\omega_2 = \sqrt{3}\omega_D$, in which case $C_p = \frac{1}{2}C_0$, $d_1 = 1.5$ and $d_2 = 3$, and Q_M is equal to

$$(Q_M)_{\min} = \frac{2}{\gamma}\sqrt[4]{\frac{27}{4}}\sqrt{\frac{\omega_1}{\omega_D}}. \quad (32)$$

The corresponding optimum idler load resistance is

$$(R_L)_{\text{opt}} = \frac{1}{q_{\text{opt}}} \frac{\gamma}{2\omega_1 C_0} \sqrt[4]{\frac{4}{27}} \sqrt{\left(\frac{\omega_1}{\omega_D}\right)^3}. \quad (33)$$

For maximum bandwidth at arbitrary R_L the self-resonant frequency of the variable capacitance should thus be as large as possible, and the idler frequency of the amplifier should be approximately equal to $\sqrt{3}\omega_D$. According to Fig. 8 the optimum idler frequency is, however, not critical, and nearly optimum bandwidth is obtained for $1.25\omega_D < \omega_2 < 2.5\omega_D$.

Although it has not been shown that the limitation $d_1 d_2 \geq 4$ is valid for any nondegenerate amplifier, no single-diode configuration has yet been shown to yield a smaller value of $d_1 d_2$. However, it has been shown that the limitation $d_1 d_2 \geq 4$ is not valid in case of a balanced

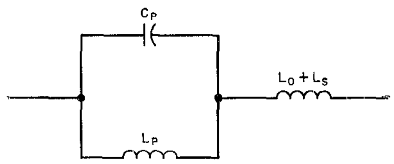


Fig. 7—Tuning element T required for resonating C_0 at ω_1 and ω_2.

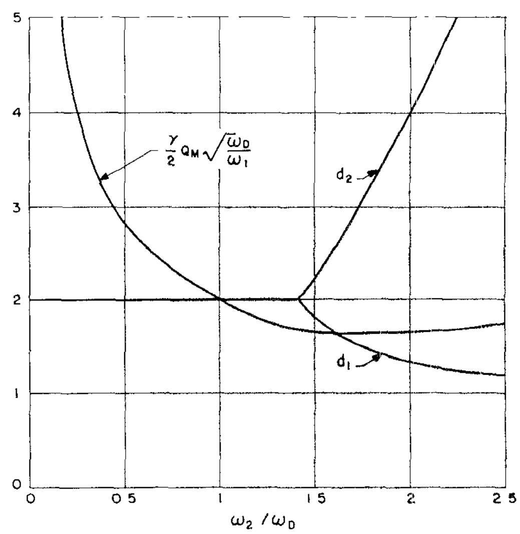


Fig. 8—Optimum Q_M, d_1 and d_2 as function of ω_2/ω_D (approximately).

parametric amplifier employing two diodes since the tuning capacitance C_p is not necessary in this configuration.[7] This means that the limiting bandwidth of a balanced parametric amplifier is theoretically twice as large as that of an unbalanced amplifier.

V. Influence of Diode Losses

At microwave frequencies $C(t)$ usually consists of a reversed-bias semiconductor diode. The main loss element of a diode is a resistance R_s in series with $C(t)$. A convenient figure of merit is the dynamic quality factor[8]

$$\tilde{Q}_1 = \frac{\gamma}{2\omega_1 C_0 R_s} \cdot \tag{34}$$

In the amplifier R_s appears as a series resistance in both the signal and idler circuit. In the idler circuit, R_s forms part of the idler load resistance R_L, which can be written

$$R_L = R_L' + R_s \tag{35}$$

where R_L' is an idler load resistance external to the diode. The parameter q then reduces to

$$q = \frac{\tilde{Q}_1}{1 + R_L'/R_S} \sqrt{\frac{d_2}{d_1}\left(\frac{\omega_1}{\omega_2}\right)^3}. \tag{36}$$

In the signal circuit R_s causes a reduction of the limiting bandwidth as compared to the limiting bandwidth of a corresponding lossless diode at the same R_L. From (7), the degree of this reduction (if $q \approx 1$) is mainly determined by the ratio R_A/R_s. Assuming

$$\frac{R_A}{R_s} = \frac{\omega_1}{\omega_2} \frac{\tilde{Q}_1^2}{1 + R_L'/R_s} \gg 1 \tag{37}$$

and using (36), the previous results are approximately valid in the case of a lossy diode. For a high-quality diode (37) is usually true, since (37) according to (42) is also necessary for obtaining a low-noise temperature of the amplifier.

Assuming in the following that (37) is true, with maximum bandwidth at a given ω_2/ω_1, we obtain

$$(1 + R_L'/R_s)_{\text{opt}} = \frac{\tilde{Q}_1}{q_{\text{opt}}} \sqrt{\frac{d_2}{d_1}\left(\frac{\omega_1}{\omega_2}\right)^3} \tag{38}$$

or, if $R_L' = 0$,

$$(\tilde{Q}_1)_{\text{opt}} = q_{\text{opt}} \sqrt{\frac{d_1}{d_2}\left(\frac{\omega_2}{\omega_1}\right)^3}. \tag{39}$$

At optimum R_L', assuming $\tilde{Q}_1 > (\tilde{Q}_1)_{\text{opt}}$, the condition (37) reduces to

$$\frac{\tilde{Q}_1}{(\tilde{Q}_1)_{\text{opt}}} q_{\text{opt}}^2 \frac{d_1}{d_2}\left(\frac{\omega_2}{\omega_1}\right)^2 \gg 1 \tag{40}$$

where $(\tilde{Q}_1)_{\text{opt}}$ is given by (39). It follows that if $(\omega_2/\omega_1)^2$ is large, and if the external idler load resistance is adjusted for maximum bandwidth, the bandwidth is little affected by the diode loss resistance, and can be improved little for $\tilde{Q}_1 > (\tilde{Q}_1)_{\text{opt}}$ (unless in the case $\omega_2 \gg \omega_D$ the ratio d_2/d_1 is appreciable). If $(\omega_2/\omega_1)^2$ is large it is thus generally advantageous to choose $\tilde{Q}_1 = (\tilde{Q}_1)_{\text{opt}}$ and to use no external idler load resistance, which simplifies the amplifier construction.

According to Section IV maximum bandwidth is obtained if $\omega_2 \cong \sqrt{3}\omega_D$, and according to (33), (34) and (35), if $\tilde{Q}_1$ is equal to

$$(\tilde{Q}_1)_{\text{opt}} = q_{\text{opt}} \sqrt[4]{\frac{27}{4}} \sqrt{\left(\frac{\omega_D}{\omega_1}\right)^3} \tag{41}$$

assuming $R_L' = 0$. If the available quality factor of the diode is less than (41), maximum bandwidth is generally reached at a lower idler frequency and is limited by $\tilde{Q}_1$.

VI. Noise Temperature

Due to thermal noise of R_s and R_L' the effective input noise temperature T_A of the amplifier in a high gain approximation is

$$T_A = \left\{\frac{\omega_1}{\omega_2} + \frac{1 + \omega_1/\omega_2}{(\omega_1/\omega_2)\tilde{Q}_1^2/(1 + R_L^1/R_s) - 1}\right\} T_D \tag{42}$$

where T_D is the temperature of R_s and R_L'.[8] In general, for minimum noise temperature somewhat different values of ω_2/ω_1, $\tilde{Q}_1$, and R_L' are optimum than for maximum bandwidth. A compromise is therefore necessary. As shown by an example in Section VIII, good bandwidth and noise performance can simultaneously be obtained in practice.

VII. Amplifier Configurations

A practical broad-band amplifier circuit for the case $(\omega_2/\omega_1)^2 \gg 1$, is given in Fig. 9(a). The circuit is based on that of Fig. 1, with the diode D resonated at ω_1 and ω_2 by L_p, C_p, and L_s as described in Section IV. The filter F_1 of Fig. 1 is replaced by a short circuit, so that the idler load resistance is provided entirely by the diode series resistance. For broad-band operation, the circuit is preceded by a network N consisting of several resonators connected alternately in series and in parallel. The output capacitance of N fulfills essentially the role of filter F_2 in Fig. 1.

An alternative circuit, having very nearly the same properties but a slightly unsymmetric frequency characteristic, is given in Fig. 9(b). Since the idler circuit is better decoupled from the signal circuit, this circuit is more suitable in practice. Complete decoupling can be obtained if L_p is paralleled by a small capacitance C_p', so that L_p and C_p' are resonant at ω_2. For $\omega_2 > \sqrt{2}\omega_D$ approximately $(L_s)_{\text{opt}} = 0$, so that C_p reduces to a parallel

[8] K. Kurokawa, "On the use of passive circuit measurements for the adjustment of variable capacitance amplifiers," *Bell Sys. Tech. J.*, vol. 41, pp. 361–381; January, 1962.

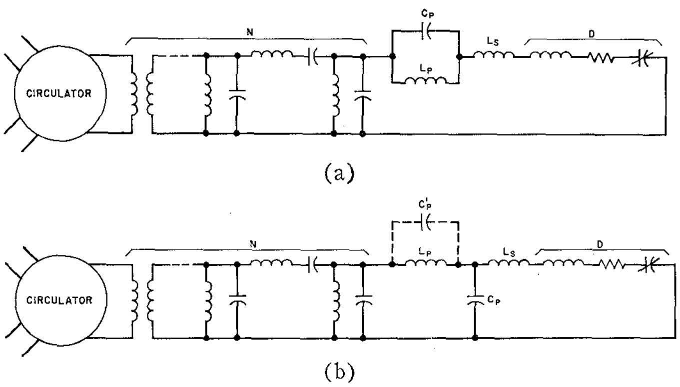


Fig. 9—Two practical amplifier circuits. Pump sources have been omitted.

capacity to the diode, which can be easily obtained in practice (Section VIII). Usually C_p is already partly present in the form of a parasitic "case" capacitance of the diode.

At microwave frequencies the use of reactances consisting of certain lengths of transmission line instead of lumped components is more practical. Due to the distributed circuit capacitance of transmission lines, in general it is found that the slope factors of circuits employing transmission lines are larger than those of comparable circuits employing lumped components. As a rule, the amplifier circuit should resemble the corresponding optimum lumped component circuit as closely as possible.

VIII. *L*-Band Amplifier

An *L*-band amplifier at a center signal frequency of 1.3 kMc was built, using the amplifier configuration of Fig. 9(b).

As shown in Fig. 10 the two conductors holding the diode overlap to form a nearly lumped capacitance C_p. The resulting idler frequency is equal to the parallel resonance frequency of the holder including the diode, whose specifications were approximately

$$C_0 = 0.7 \text{ pf}$$
$$L_0 = 0.8 \text{ nh}$$
$$\tilde{Q}_1 = 30.$$

The obtained idler frequency was approximately 10 kMc at $C_p \approx C_0$. According to (31) and Fig. 8 this idler frequency is close to the optimum idler frequency for maximum bandwidth at a nominal self-resonant frequency of the diode of 6.7 kMc.

A simplified sketch of the amplifier is given in Fig. 11. The signal circuit consists of the described diode holder in series with a coaxial line shorted at its end. The part of the line below the diode holder is a quarter wavelength long at the idler frequency and forms an idler rejection filter. The lower part of the signal cavity is sharply resonant at the pump frequency of 11.3 kMc. Pump power is fed to the diode through a reduced height *X*-band waveguide, a coaxial to waveguide

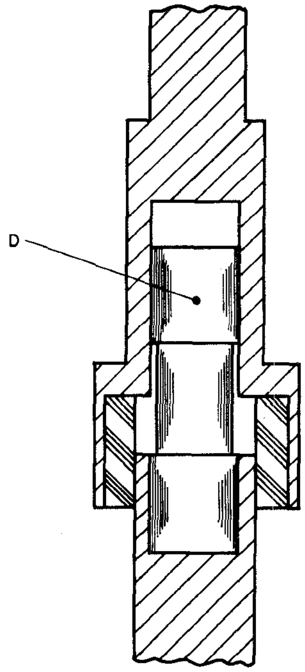


Fig. 10—Diode holder containing diode *D*.

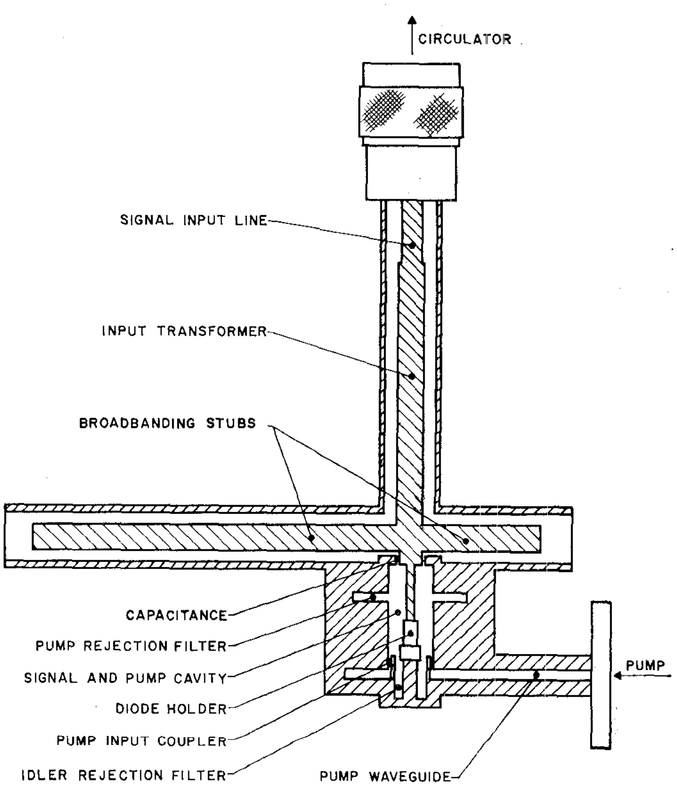


Fig. 11—*L*-band amplifier.

transition, and a short low-impedance coaxial line which prevents high-frequency components from leaking into the waveguide. A small lumped capacitance at the end of the signal cavity has a similar role. The signal broad-banding filter consists of two open stubs paralleling the input of the single-tuned amplifier. The length of one stub is somewhat shorter and the length of the other stub somewhat longer than a quarter wavelength at the center signal frequency. At moderate gain this filter, in conjunction with the input impedance transformer, provides a triple-tuned frequency response. The optimum impedance and length of the stubs was determined mainly by experiment.

According to (27) and (31) for the lumped element prototype at $C_p = C_0$, $d_1 \approx d_2 \approx 2$. Because of the distributed circuit capacitances a better estimate for the actual circuit is $d_1 = d_2 = 2.5$. Then from (13) $Q_M = 3.6$ (assuming $\gamma = 0.5$), and according to (36) $q = 1.45$. This value of q according to Fig. 6 is close to the optimum value of q for maximum bandwidth at moderate gain. From (40) it follows that the diode losses influence the bandwidth negligibly and that no significant improvement in bandwidth can be obtained by using a higher diode $\tilde{Q}_1$ and an external idler load resistance. Some calculated bandwidths obtained from Figs. 4 and 5 are given in Table I.

According to (42), the expected effective noise temperature of the amplifier is $T_A = 0.14 T_D$.

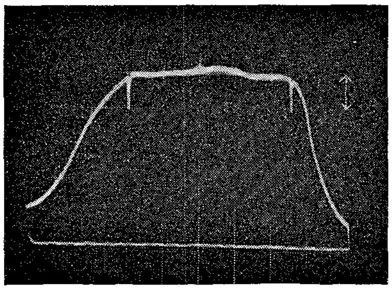
TABLE I
CALCULATED AND MEASURED BANDWIDTHS OF L-BAND AMPLIFIER

Gain	Single-tuned; 3-db bandwidth		Broad-banded; Flat Bandwidth	
	Calculated	Measured	Calculated Limit	Measured
7 db	15 per cent	15 per cent	44 per cent	23 per cent
13	6.1	5.7	30	14
20	2.6	2.3	21	8.5

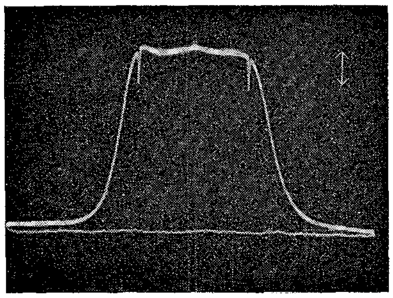
IX. EXPERIMENTAL RESULTS

In Fig. 12, pictures of some measured pass bands of the broad-band amplifier, obtained by using a square law detector, are given at various gains. In Table I the measured bandwidths are tabulated, together with the measured 3-db single-tuned bandwidths obtained by leaving away the broad-banding filters of the amplifier. At each gain the signal input transformer (Fig. 11) was readjusted so that the pumping conditions of the diode were about the same. Without transformation the gain of the amplifier was about 6 db.

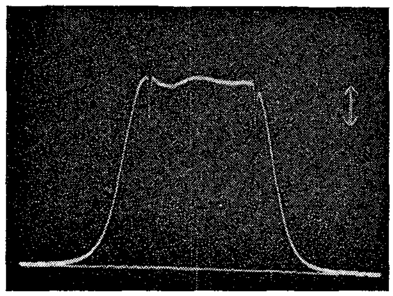
Much care was used to make the pass band as flat as possible. The obtained ripples were generally below 0.3 db peak-to-peak. Apart from careful tuning this re-



(a)



(b)



(c)

Fig. 12—Measured pass bands of the amplifier at a center frequency of 1300 Mc and a gain g. A gain of 1 db is indicated by the arrows in the pictures. Markers indicate the frequency difference Δf. (a) $g = 7$ db, $\Delta f = 300$ Mc. (b) $g = 13$ db, $\Delta f = 180$ Mc. (c) $g = 20$ db, $\Delta f = 110$ Mc.

quired the use of a certain optimum electrical line length between amplifier and circulator. The VSWR of the employed circulator was less than 1.15 in a 25 per cent frequency band. Also, care was necessary to avoid spurious resonances of the circuit at unwanted frequencies which are converted by the pump frequency or its harmonics to the signal frequency, and which manifest themselves as small peaks or dips in the pass band which, for example, depend on the setting of an external pump tuner. For stability these resonances should, of course, also be avoided. Isolation of the amplifier cavity from pump and signal circuit by capacitive discontinuities, as described in Section VIII, prevented these resonances effectively. The required pump power of the amplifier was less than 50 mw.

As shown in Table I, the measured single-tuned bandwidths compare favorably with the expected bandwidths. The obtained flat bandwidths are about 40 to

50 per cent of the theoretically limiting flat bandwidths. The bandwidth could have been widened by using a more complicated filter, but because of the small obtainable improvement and the increasing complexity, this was not attempted. Probably the bandwidth was also somewhat limited by the output impedance of the circulator.

A disadvantage of a broad-band parametric amplifier is the sensitivity of the gain at off-center frequencies for pump power changes. Since the circuit is highly compensated, the pass band changes considerably if the amplifier becomes detuned by a change of the mean capacitance of the diode caused by a change of pump power. It is advisable not to run the amplifier at a high gain.

The measured effective input noise temperature of the amplifier is about 70°K at room temperature. An earlier version of the amplifier working at the temperature of liquid nitrogen and employing a GaAs diode has shown a minimum noise temperature of 29°K. Due to circuit losses these noise temperatures are somewhat higher than expected.

X. Conclusions

The single-tuned and limiting flat bandwidth of a nondegenerate diode parametric amplifier of the reflection type having broad-banding resonators in the signal circuit and a single-tuned idler circuit has been calculated. The limiting flat bandwidth is about 75 per cent of that of an amplifier having broad-banding resonators in both signal and idler circuit. The difference usually does not justify the use of external broad-band filters in the idler circuit.

The bandwidth limitation of some practical lumped element amplifier circuits has been calculated. It has been shown that the optimum idler circuit consists of a capacitance in parallel with the diode, and that the optimum idler frequency is approximately $\sqrt{3}$ times the self-resonance frequency of the diode, assuming that the losses of the diode are small.

A simple triple-tuned L-band amplifier at a signal frequency of 1.3 kMc and an idler frequency of 10 kMc has been described. A nearly flat bandwidth of 23 per cent at 7 db gain, and an effective input noise temperature of 70°K at room temperature and of 29°K at the temperature of liquid nitrogen (77°K) has been obtained. This makes the amplifier suitable as a low-noise, wide-band amplifier for some applications, for example, in radio astronomy.

By reducing the capacitance and parasitic lead inductance of the diode, *e.g.*, by incorporating the idler tuning capacitance in the diode package itself, it should be possible to realize the same design at appreciably higher signal and idler frequencies.

A problem remaining to be solved is the calculation of the bandwidth for a finite filter network in the signal circuit and the design of that filter network.

Acknowledgment

The described work was performed and reported in internal memoranda by Mr. DeJager while at the Bell Telephone Laboratories, Murray Hill, N. J., from 1961 to 1963. The manuscript was submitted for publication posthumously by his former colleagues as a tribute to Jan T. DeJager's significant contributions in the field of parametric amplification.